Differential geometric and group theoretic methods: selected topics

1. Differentiable manifolds, smooth functions and mappings, diffeomorphisms.

2. Tangent vector, tangent bundle, tangent map (differential of smooth map).

3. Tensors, tensor bundles, smooth tensor fields. Pull-back and push-forward operations on tensor fields: definitions and local formulas.

4. Algebra of exterior forms, differential forms, exterior derivation.

5. Algebraic definition of the Lie derivative of tensor fields, the explicit local formula of the Lie derivative.

6. Connection between vector fields and local flows, `dynamical' definition of the Lie derivative, the equivalence of the algebraic and dynamical definition.

7. Identities for the exterior, inner and Lie derivatives of differential forms.

8. Lie group and its Lie algebra, exponential map. Connection between homomorphisms of Lie groups and their Lie algebras. Action of a Lie group and its infinitesimal generator.

9. Symplectic manifold, Hamiltonian vector field and Poisson bracket on a symplectic manifold.

10. Cotangent bundles as symplectic manifolds. Darboux theorem (canonical coordiantes) in the case of symplectic vector spaces and symplectic manifolds.

11. Poisson manifolds and Poisson maps. Hamiltonian system and their symmetries. Duals of Lie algebra as Poisson manifolds, and their symplectic leaves.

12. Riemann and pseudo-Riemann manifolds, Hamiltonian description of geodesics. Magnetic cotangent bundles.

13. Symplectic and Hamiltonian actions of Lie groups, the momenttum map, examples (momentum maps associated with point transformations and with the coadjoint action).

14. The ideology of Hamiltonian symmetry reduction, the classical Marsden-Weinstein reduction.

Recommended literature

J.E. Marsden, T.S. Ratiu: Introduction to mechanics and symmetry, second edition, Springer-Verlag, 1999.

B.A. Dubrovin, A. T. Fomenko, S. P. Novikov: Modern geometry - methods and applications, Vols. I and II, Springer-Verlag, 1985.

T. Frankel: The geometry of physics, second edition, Cambridge University Press, 2004.